

# Measurement of electrical power

*Instantaneous, mean, active, reactive and apparent electrical power, power factor, etc.*

*We would like to remind you about these basic parameters in electronics and about three-phase measurement methods.*

## Definition of electrical power

At a given moment, when a current **i** travels from generator **G** to receiver **R** in the direction defined by the voltage **v** delivered by the generator (figure 1), the **instantaneous power** supplied to the receiver R is equal to **product v·i**.

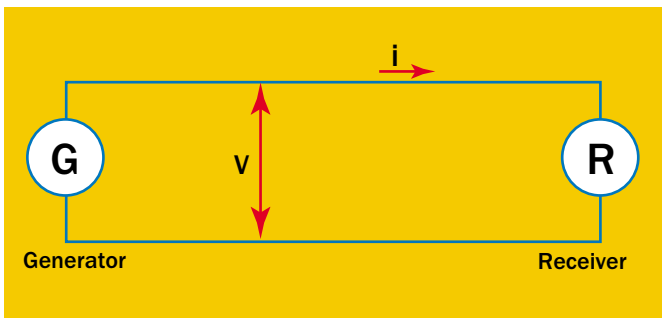


figure 1

If the voltage and current are DC, the mean power  $V \cdot I$  is equal to the instantaneous power  $v \cdot i$ .

If the voltage and current are **sinusoidal AC**, there is generally a **phase shift**  $\varphi$  between the voltage and the current (figure 2).

The **instantaneous values** of voltage **v** and current **i** have the form:

$$v = V_{\max} \cos \omega t$$

$$i = I_{\max} \cos (\omega t - \varphi)$$

Where  $\omega$ , the pulse, is proportional to the frequency  $F$  ( $\omega = 2\pi F$ ).

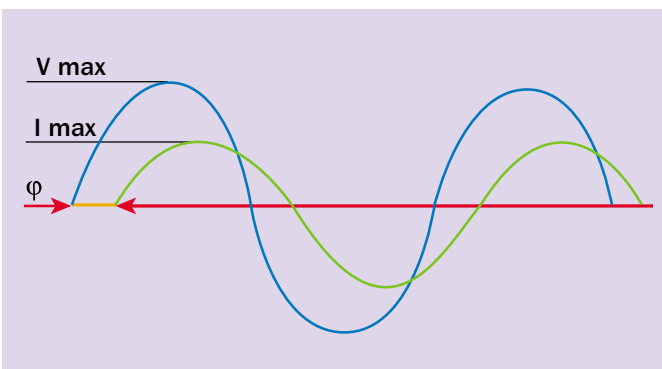


figure 2

The **phase shift**  $\varphi$  is, conventionally, counted as positive when the current is delayed in relation to the voltage.

The **instantaneous power** has a value of:  $V_{\max} \cdot I_{\max} \cdot \cos \omega \cdot \cos (\omega t - \varphi)$ . You must take the average value of this product during a period to obtain the expression of the power provided by generator G to receiver R. This power is called the **active power** and is expressed by the formula:

$$P = \frac{V_{\max} \cdot I_{\max}}{\sqrt{2}} \cos \varphi = V_{\text{eff}} \cdot I_{\text{eff}} \cdot \cos \varphi$$

The **wattmeters** provide the expression of this product, either by causing a deviation of the pointer in the case of a device with an electrodynamic or ferrodynamic moving coil, or by supplying a DC current or a voltage

proportional to the product in the case of electronic wattmeters; this current or this voltage is then applied to an analogue or digital display.

The existence of a phase shift  $\varphi$  between the current and the voltage leads, for AC currents, to the introduction of 3 additional quantities:

- **The apparent power  $S = V_{\text{eff}} \cdot I_{\text{eff}}$** , in VA (volt-amperes), defining the voltage  $V_{\text{eff}}$  not to be exceeded (insulator breakdown, increase in core loss) and the intensity  $I_{\text{eff}}$  circulating in the receivers.

- **The power factor:**

$$\cos \varphi = \frac{P}{S} = \frac{P}{V_{\text{eff}} \cdot I_{\text{eff}}}$$

when the current and voltage are sinusoidal quantities.

- **The reactive power  $Q = V_{\text{eff}} \cdot I_{\text{eff}} \cdot \sin \varphi$** , in rva (reactive volt-amperes). The latter may be directly measured by a wattmeter if for voltage  $V_{\max} \cdot \cos \omega t$  we substitute a phase-shifted voltage of  $\pi/2$ , i.e.  $V_{\max} \times \cos (\omega t - \pi/2)$ .

The mean product measured will be

$V_{\max} \cdot I_{\max} \cdot \cos (\omega t - \pi/2) \times \cos (\omega t - \varphi)$  which is expressed by:

$$Q = \frac{V_{\max} \cdot I_{\max}}{\sqrt{2}} \cos (\pi/2 - \varphi) = V_{\text{eff}} \cdot I_{\text{eff}} \cdot \sin \varphi$$

Knowing P and Q, we can calculate the apparent power and the power factor:

Apparent power:  $S = \sqrt{P^2 + Q^2}$

Power factor:  $PF = P/S = P/\sqrt{P^2 + Q^2}$

Knowing the parameters defined above: active power, reactive power, apparent power, power factor, is fundamental in electrical engineering and enables accurate calculation of the characteristics of the equipment used: yield, load,  $\cos \varphi$ , utilisation limits. The wattmeters used for these measurements are classified in three major families: electrodynamic, ferrodynamic and electronic.

## Measurement of active power

### 4-wire balanced three-phase measurement (3 phases + neutral)

The intensities circulating in the three phases are equal in terms of rms values  $I_1 = I_2 = I_3$  and show the same phase shift  $\varphi$  in relation to the respective voltages of the 3 phases.

If  $U_{1N}$  is the simple voltage measured between phase 1 and neutral, power  $P_1$  supplied by phase 1 will be obtained by connecting a wattmeter as shown in figure 3.

Its value will be:  $P_1 = U_{1N} \cdot I_1 \cdot \cos \varphi$

The total power supplied P will be equal to  $3 P_1$ .

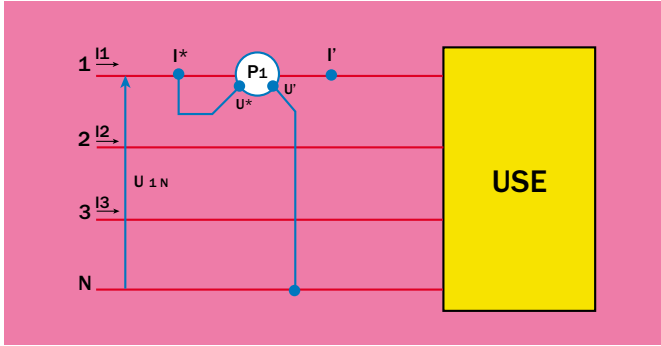


figure 3

Note: The expression  $P_1 = U_{1N} \cdot I_1 \cdot \cos \varphi$  is the scalar product of the 2 vectors

$\vec{U}_{1N}$  and  $\vec{I}_1$  which enables use of the notation

$$P_1 = \vec{U}_{1N} \cdot \vec{I}_1$$

and in three-phase:

$$P = \vec{U}_{1N} \cdot \vec{I}_1 + \vec{U}_{2N} \cdot \vec{I}_2 + \vec{U}_{3N} \cdot \vec{I}_3$$

**Measurement in 3-wire balanced three-phase (3 phases no neutral)**

The intensities circulating in the three phases are equal  $I_1 = I_2 = I_3$ . An artificial neutral is created using three resistors R, R et R'. The sum  $R' + r$  must be equal to R (r is the resistance of the voltage circuit of the unit).

This returns us to the previous case with  $U_{1N}$  between phase 1 and the artificial neutral (figure 4).

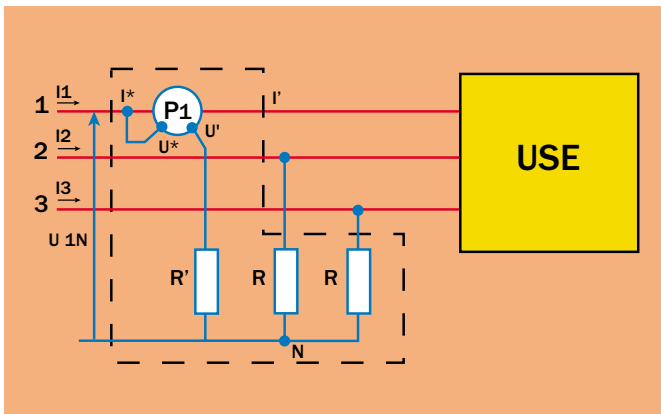


figure 4

$P_1$  = Power supplied on phase 1

Totale P supplied =  $3 U_{1N} \cdot I_1 \cdot \cos \varphi = 3P_1$ .

With many wattmeters, the balanced three-phase measurements (3 phases no neutral) are performed directly; the artificial neutral point recreated by the resistors R, R and R' is included in the instrument (astatic wattmeter, CdA 778 wattmeter, for example). This design is shown in the diagram by the dotted section.

**Measurement in 3-wire unbalanced three-phase (3 phases no neutral) - method using two wattmeters.**

Whether the circuit is **balanced** or **not in the absence of a neutral, there remains  $I_1 + I_2 + I_3 = 0$ .**

In this case, the general expression of the power given above is simplified

$$P = (\vec{U}_{1N} - \vec{U}_{3N}) \cdot \vec{I}_1 + (\vec{U}_{2N} - \vec{U}_{3N}) \cdot \vec{I}_2$$

$$\text{so } P = \vec{U}_{13} \cdot \vec{I}_1 + \vec{U}_{23} \cdot \vec{I}_2$$

and the measurement of the total power may be carried out using two wattmeters (figure 5).

$U_{13}$  and  $U_{23}$  are the phase-to-phase voltages measured respectively between phase 1 and phase 3 and then between phase 2 and phase 3.

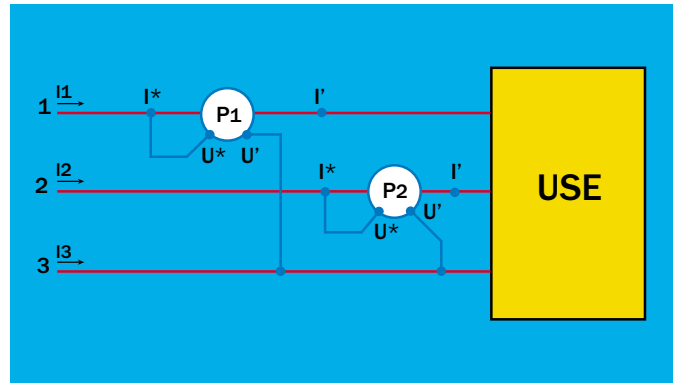


figure 5

Two cases may arise:

a)  $P_1 \geq 0$  and  $P_2 \geq 0$ , then  $P_{total} = P_1 + P_2$

b) one wattmeter deviates to the right and the other is as far as it will go to the left. To read the second; transfer the feed wires to the voltage circuit:  $U^* \cdot U'$  becomes  $U' \cdot U^*$ .

The value will be considered negative and we will obtain:  $P_{total} = P_1 - P_2$

If it is a digital wattmeter we will add together the algebraic values displayed.

Note: it is possible to use a single wattmeter successively connected to 2 positions, using an inverter switch. This type of switch contains auxiliary contacts ensuring short-circuiting of the unused contacts.

**Measurement in 4-wire balanced three-phase (3 phases + neutral)**

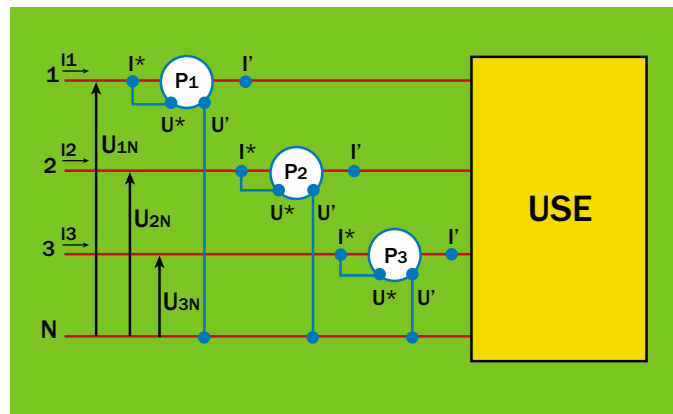


figure 6

We obtain  $P_{total} = P_1 + P_2 + P_3$  (figure 6).

In this case, we must use 3 wattmeters and add the readings together. If the measurement is stable, we can successively carry out 3 measurements with a single wattmeter. Caution: it is recommended to use a system preventing the intensity circuits from being cut off during switching.