Measurement of electrical power

Instantaneous, mean, active, reactive and apparent electrical power, power factor, etc.

We would like to remind you about these basic parameters in electronics and about three-phase measurement methods.

Definition of electrical power

At a given moment, when a current \( i \) travels from generator \( G \) to receiver \( R \) in the direction defined by the voltage \( v \) delivered by the generator (figure 1), the instantaneous power supplied to the receiver \( R \) is equal to product \( v \cdot i \).

If the voltage and current are DC, the mean power \( V \cdot I \) is equal to the instantaneous power \( v \cdot i \).

If the voltage and current are sinusoidal AC, there is generally a phase shift \( \phi \) between the voltage and the current (figure 2).

The instantaneous values of voltage \( v \) and current \( i \) have the form:

\[
\begin{align*}
  v & = V_{\text{max}} \cos \omega t \\
  i & = I_{\text{max}} \cos (\omega t - \phi)
\end{align*}
\]

Where \( \omega \), the pulse, is proportional to the frequency \( F \) (\( \omega = 2\pi F \)).

The phase shift \( \phi \) is, conventionally, counted as positive when the current is delayed in relation to the voltage.

The instantaneous power has a value of: \( V_{\text{max}} \cdot I_{\text{max}} \cdot \cos \omega t \cdot \cos (\omega t - \phi) \).

You must take the average value of this product during a period to obtain the expression of the power provided by generator \( G \) to receiver \( R \). This power is called the active power and is expressed by the formula:

\[
P = \frac{V_{\text{max}} \cdot I_{\text{max}} \cdot \cos \phi}{\sqrt{2}} = V_{\text{rms}} \cdot I_{\text{rms}} \cdot \cos \phi
\]

The wattmeters provide the expression of this product, either by causing a deviation of the pointer in the case of a device with an electrodynamic or ferrodynamic moving coil, or by supplying a DC current or a voltage proportional to the product in the case of electronic wattmeters; this current or this voltage is then applied to an analogue or digital display.

The existence of a phase shift \( \phi \) between the current and the voltage leads, for AC currents, to the introduction of 3 additional quantities:

- The apparent power \( S = V_{\text{rms}} \cdot I_{\text{rms}} \) in VA (volt-amperes), defining the voltage \( V_{\text{rms}} \) not to be exceeded (insulator breakdown, increase in core loss) and the intensity \( I_{\text{rms}} \) circulating in the receivers.

- The power factor:

\[
\cos \phi = \frac{P}{S} = \frac{P}{V_{\text{rms}} \cdot I_{\text{rms}}}
\]

where the current and voltage are sinusoidal quantities.

- The reactive power \( Q = V_{\text{rms}} \cdot I_{\text{rms}} \cdot \sin \phi \), in rva (reactive volt-amperes).

The latter may be directly measured by a wattmeter if for voltage \( V_{\text{rms}} \cdot \cos \omega t \) we substitute a phase-shifted voltage of \( \pi/2 \), i.e. \( V_{\text{rms}} \cdot \cos (\omega t - \pi/2) \).

The mean product measured will be

\[
V_{\text{rms}} \cdot I_{\text{rms}} \cdot \cos (\pi/2 - \phi) \cdot \cos (\omega t - \phi)
\]

which is expressed by:

\[
Q = V_{\text{rms}} x I_{\text{rms}} \cdot \sin \phi \sqrt{2}
\]

Knowing \( P \) and \( Q \), we can calculate the apparent power and the power factor:

Apparent power:

\[
S = \sqrt{P^2 + Q^2}
\]

Power factor:

\[
\text{PF} = \frac{P}{S} = \frac{P}{\sqrt{P^2 + Q^2}}
\]

Knowing the parameters defined above: active power, reactive power, apparent power, power factor, is fundamental in electrical engineering and enables accurate calculation of the characteristics of the equipment used: yield, load, cos \( \phi \), utilisation limits. The wattmeters used for these measurements are classified in three major families: electrodynamic, ferrodynamic and electronic.

Measurement of active power

4-wire balanced three-phase measurement (3 phases + neutral)

The intensities circulating in the three phases are equal in terms of rms values \( I_1 = I_2 = I_3 \) and show the same phase shift \( \phi \) in relation to the respective voltages of the 3 phases.

If \( U_{\text{rms}} \) is the simple voltage measured between phase 1 and neutral, power \( P \) supplied by phase 1 will be obtained by connecting a wattmeter as shown in figure 3.

Its value will be: \( P_1 = U_{\text{rms}} \cdot I_1 \cdot \cos \phi \)

The total power supplied \( P \) will be equal to 3 \( P_1 \).
Note: The expression \( P_1 = U_{1N} \cdot I_1 \cdot \cos \varphi \) in the scalar product of the 2 vectors \( U_{1N} \) and \( I_1 \), which enables use of the notation \( P_1 = U_{1N} \cdot I_1 \)
and in three-phase:
\[ P = U_{1N} \cdot I_1 + U_{2N} \cdot I_2 + U_{3N} \cdot I_3 \]

**Measurement in 3-wire balanced three-phase (3 phases no neutral)**

The intensities circulating in the three phases are equal \( I_1 = I_2 = I_3 \).
An artificial neutral is created using three resistors \( R, R \) et \( R' \). The sum \( R' + r \) must be equal to \( R \) (\( r \) is the resistance of the voltage circuit of the unit).

This returns us to the previous case with \( U_{1N} \) between phase 1 and the artificial neutral (figure 4).

\[ P_1 = \text{Power supplied on phase 1} \]
\[ \text{Totale } P \text{ supplied} = 3 \cdot U_{1N} \cdot I_1 \cdot \cos \varphi = 3P_1. \]

With many wattmeters, the balanced three-phase measurements (3 phases no neutral) are performed directly; the artificial neutral point recreated by the resistors \( R, R \) and \( R' \) is included in the instrument (astatic wattmeter, CdA 778 wattmeter, for example). This design is shown in the diagram by the dotted section.

**Measurement in 3-wire unbalanced three-phase (3 phases no neutral) - method using two wattmeters.**

Whether the circuit is balanced or not in the absence of a neutral, there remains \( I_1 + I_2 + I_3 = 0 \).

In this case, the general expression of the power given above is simplified
\[ P = (U_{1N} - U_{3N}) \cdot I_1 + (U_{2N} - U_{3N}) \cdot I_2 \]
so \( P = U_{1N} \cdot I_1 + U_{2N} \cdot I_2 \)
and the measurement of the total power may be carried out using two wattmeters (figure 5).

\( U_{1N} \) and \( U_{2N} \) are the phase-to-phase voltages measured respectively between phase 1 and phase 3 and then between phase 2 and phase 3.

Two cases may arise:
a) \( P_1 \geq 0 \) and \( P_1 \geq 0 \), then \( P_{\text{tot}} = P_1 + P_2 \)
b) one wattmeter deviates to the right and the other is as far as it will go to the left. To read the second; transfer the feed wires to the voltage circuit: \( U^* \cdot U' \) becomes \( U' \cdot U^* \).
The value will be considered negative and we will obtain: \( P_{\text{tot}} = P_1 - P_2 \)

If it is a digital wattmeter we will add together the algebraic values displayed.

Note: it is possible to use a single wattmeter successively connected to 2 positions, using an inverter switch. This type of switch contains auxiliary contacts ensuring short-circuiting of the unused contacts.

**Measurement in 4-wire balanced three-phase (3 phases + neutral)**

We obtain \( P_{\text{tot}} = P_1 + P_2 + P_3 \) (figure 6).

In this case, we must use 3 wattmeters and add the readings together. If the measurement is stable, we can successively carry out 3 measurements with a single wattmeter. Caution: it is recommended to use a system preventing the intensity circuits from being cut off during switching.